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LETTERS TO THE EDITOR

A FURTHER NOTE ON THE "DYNAMIC ANALYSIS OF GENERALLY SUPPORTED BEAMS USING FOURIER SERIES"

M. J. Maurizi and G. G. Robledo
Department of Engineering, *Universidad Nacional del Sur*, 8000 *Bahía Blanca*, *Argentina*

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The authors wish to congratulate Drs. Wang and Lin for their thorough and very interesting study [1]. The published paper has two contributions to offer. These are:

(1) The applicability of Fourier series to the dynamic analysis of beams having arbitrary boundary conditions.

(2) The procedure can be extended to analyze the dynamic response of two-dimensional plates having various combinations of supporting conditions and subjected to any time dependent load distributions.

With regard to previous works in which the Fourier series solution is applied in the dynamic analysis of elastic structures, other references [2–4] should be cited, as follows. In 1976 Greif and Mittendorf [2] presented a method for vibration analysis of a wide class of beam, plate and shell problems including the effects of variable geometry and material properties. The method is based on the discrete technique of component mode analysis. For each of these components the mode shapes are written in terms of Rayleigh–Ritz expansions involving simple Fourier sine or cosine series. However, it is essential to employ Stokes' transformation to differentiate these series properly and to release those geometric boundary conditions that have been unavoidably forced in due to the nature of the modal expansion [5, 6]. Continuity conditions between substructures are enforced by appropriate Lagrange multipliers.

The resulting frequency determinant is an exact mathematical expression for the natural frequencies of the system and the associated eigenvector is mixed in the sense that it contains both a force (moment) and displacement type quantities. This approach, as mentioned previously, can be employed for a vibrating cylindrical shell with the use of simple Fourier series and component modes [3]. The basic technique proceeds in the same manner as outlined for beam problems although it is obviously more complicated since the shell problem involves three displacement components. Subsequently, the cited authors showed in reference [4] that a single segment guided–guided beam, coupled with a sine series, is sufficient to derive an *exact* master frequency determinant for a *general* multi-segmented beam with arbitrary material and geometric properties in each segment and general boundary conditions at the ends.

The entire procedure for finding natural frequencies for multi-segment beams based on a Fourier sine series may also be done with a cosine series expansion of the modes. The basic single element appropriate to this case is the familiar simply-supported beam and is also sufficient to derive an exact master frequency determinant for the general beam problem. Furthermore, in the Fourier series technique it is quite simple to include the effects of springs and discrete masses at intermediate points and boundaries, as well as the effects of variable geometric and material properties. In this respect, specific problems examined by Mittendorf and Greif [4] with each alternative procedure are showed in Figure 1. The more effective of these approaches is dependent on the nature of the vibration problem treated which is practically based on minimizing the size of the frequency determinant.

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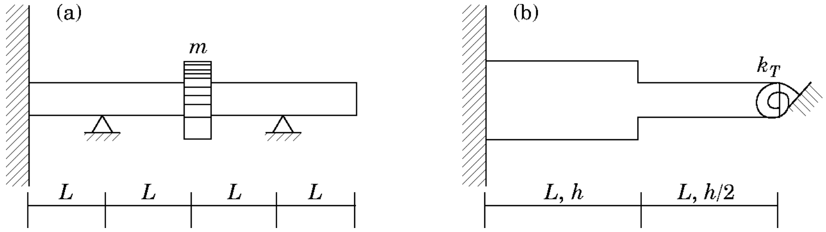

Figure 1. Systems analyzed by the Fourier series technique: (a) multi-support beam with intermediate mass m; (b) beam with discontinuous height and torsional spring at the boundary.

For the first example, the results obtained by the explained analysis and those exposed by other investigators (see for example the methodology applied by Bapat and Bapat in reference [7]) are very close together. Additionally, De Rosa *et al*. [8] examined the most general stepped system and all the results for the fundamental frequency given in reference [4], for the typical problem shown in Figure 1(b), can be obtained as a limiting case.

It is appropriate to conclude this letter by quoting Liew and Hung [9]:

"*It is indeed the primary challenge in engineering research to constantly uncover simple and yet fairly accurate solutions of contemporary engineering problems*".

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